Quiz #3

MATH 126-102 — Calculus II
Thursday, Sept 10, 2026

Please print your name:

Problem 1. Consider the region in the first quadrant bounded by the curves $y=x^2$ and $y=x$.

Use the shell method to find the volume of the solid generated by revolving this region about the y-axis.

Solution. First, make a sketch! Note that the region extends from $x=0$ to $x=1$ (if this is not clear from the sketch, you can compute the intersection of $y=x^2$ and $y=x$: $x^2=x$ has solutions $x=0,1$).

Note that, if we take vertical cross-sections, then we get cylindrical shells after revolving.

The shell at position x has radius x and height $x-x^2$. Given a thickness of $\mathrm{d}x$, it therefore has (approximate) volume $2\pi x(x-x^2)\mathrm{d}x$. Hence, the total volume is

$$\int_0^1 2\pi x(x-x^2)\mathrm{d}x = 2\pi\int_0^1 (x^2-x^3)\mathrm{d}x = 2\pi\left[\frac{1}{3}x^3-\frac{1}{4}x^4\right]_0^1 = 2\pi\left(\frac{1}{3}-\frac{1}{4}\right) = \frac{\pi}{6}.$$

Alternatively. Here is how the washer method looks like for the same problem. In this case, we consider horizontal cross-sections from $y=0$ to $y=1$ (see sketch). The washers have outer radius $\sqrt{y}$ ($y=x^2$ is equivalent to $x=\sqrt{y}$) and inner radius y. Hence, their cross-sectional area is $\pi(\sqrt{y})^2-\pi y^2=\pi(y-y^2)$. Overall, the total volume is again

$$\int_0^1 \pi(y-y^2)\mathrm{d}y = \pi\left[\frac{1}{2}y^2-\frac{1}{3}y^3\right]_0^1 = \pi\left(\frac{1}{2}-\frac{1}{3}\right) = \frac{\pi}{6}.$$

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