

Quiz #4

Please print your name:

Problem 1. A conical container of radius 10 ft and height 20 ft is completely filled with water (the tip of the cone is at the bottom). Water weighs 62.4 lb/ft³.

Below, you are being asked to write down integrals. **Do not evaluate these integrals.**

- (a) Write down an integral for how much work it will take to pump the water to a level of 5 ft above the cone's rim.
- (b) Now, suppose instead that the container is initially only filled to a height of 15 ft. Again, write down an integral for how much work it will take to pump the water to a level of 5 ft above the cone's rim.

Solution.

- (a) Make a sketch! Let us denote with y (in ft) the vertical position in such a way that the tip is at $y = 0$ and the rim is at $y = 20$. We need to pump the water to the level $y = 20 + 5 = 25$.

We consider a horizontal slice at height y and thickness dy :

- This slice needs to be lifted up $25 - y$ (ft).
- The slice is (almost) a disk with radius $r = \frac{10}{20}y = \frac{y}{2}$. Hence, its volume is $\pi r^2 dy = \frac{\pi}{4}y^2 dy$ (ft³).
- The weight of the slice is $62.4 \frac{\pi}{4}y^2 dy$ (lb) and it needs to be lifted up $25 - y$ (ft). That takes work of $(25 - y) \cdot 62.4 \frac{\pi}{4}y^2 dy$ (ft-lb).
- “Adding” up, the total work required is

$$\int_0^{20} (25 - y) \cdot 62.4 \frac{\pi}{4} y^2 dy,$$

which is about $1.31 \cdot 10^6$ ft-lb (but we were asked to not evaluate the integral).

- (b) We now only lift “slices” from height 0 to 15 ft. In that case, the total work required is

$$\int_0^{15} (25 - y) \cdot 62.4 \frac{\pi}{4} y^2 dy.$$