Quiz #4

MATH 126-102 — Calculus II
Thursday, Sept 17, 2026

Please print your name:

Problem 1. A conical container of radius 20 ft and height 40 ft is completely filled with brine (the tip of the cone is at the bottom). Brine weighs 75 lb/ft^3.

Below, you are being asked to write down integrals. **Do not evaluate these integrals.**

(a) Write down an integral for how much work it will take to pump the water to a level of 3 ft above the cone's rim.

(b) Now, suppose instead that the container is initially only filled to a height of 30 ft. Again, write down an integral for how much work it will take to pump the water to a level of 3 ft above the cone's rim.

Solution.

(a) Make a sketch! Let us denote with y (in ft) the vertical position in such a way that the tip is at $y=0$ and the rim is at $y=40$. We need to pump the water to the level $y=40+3=43$.

We consider a horizontal slice at height y and thickness $\mathrm{d}y$:

- This slice needs to be lifted up $43-y$ (ft).
- The slice is (almost) a disk with radius $r=\frac{20}{40}y=\frac{y}{2}$. Hence, its volume is $\pi r^2\,\mathrm{d}y=\frac{\pi}{4}y^2\mathrm{d}y$ (ft^3).
- The weight of the slice is $75\,\frac{\pi}{4}y^2\,\mathrm{d}y$ (lb) and it needs to be lifted up $43-y$ (ft). That takes work of $(43-y)\cdot 75\,\frac{\pi}{4}y^2\mathrm{d}y$ (ft-lb).
- "Adding" up, the total work required is

$$\int_0^{40}(43-y)\cdot 75\,\frac{\pi}{4}y^2\mathrm{d}y,$$

which is about $1.63\cdot 10^7$ ft-lb (but we were asked to not evaluate the integral).

(b) We now only lift "slices" from height 0 to 30 ft. In that case, the total work required is

$$\int_0^{30}(43-y)\cdot 75\,\frac{\pi}{4}y^2\mathrm{d}y.$$

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