

Midterm #2

Please print your name:

No notes, calculators or tools of any kind are permitted. There are 31 points in total. You need to show work to receive full credit.

Good luck!

Problem 1. (6 points) In the case $\lambda > 0$, find all eigenfunctions and eigenvalues of

$$y'' + \lambda y = 0, \quad y'(0) = 0, \quad y(3) = 0.$$

Problem 2. (3 points) Let $y(x)$ be the unique solution to the IVP $y'' = 5 + 2(x - 1)y^2$, $y(0) = 1$, $y'(0) = 2$. Determine the first several terms (up to x^3) in the power series of $y(x)$.

Problem 3. (6 points) Derive a recursive description of a power series solution $y(x)$ of the DE $y'' = (5x^2 - 3)y$.

Problem 4. (3 points) A mass-spring system is described by the DE $3y'' + ky = F(t)$ where $F(t)$ is an external force with period 5. For which values of k can resonance occur?

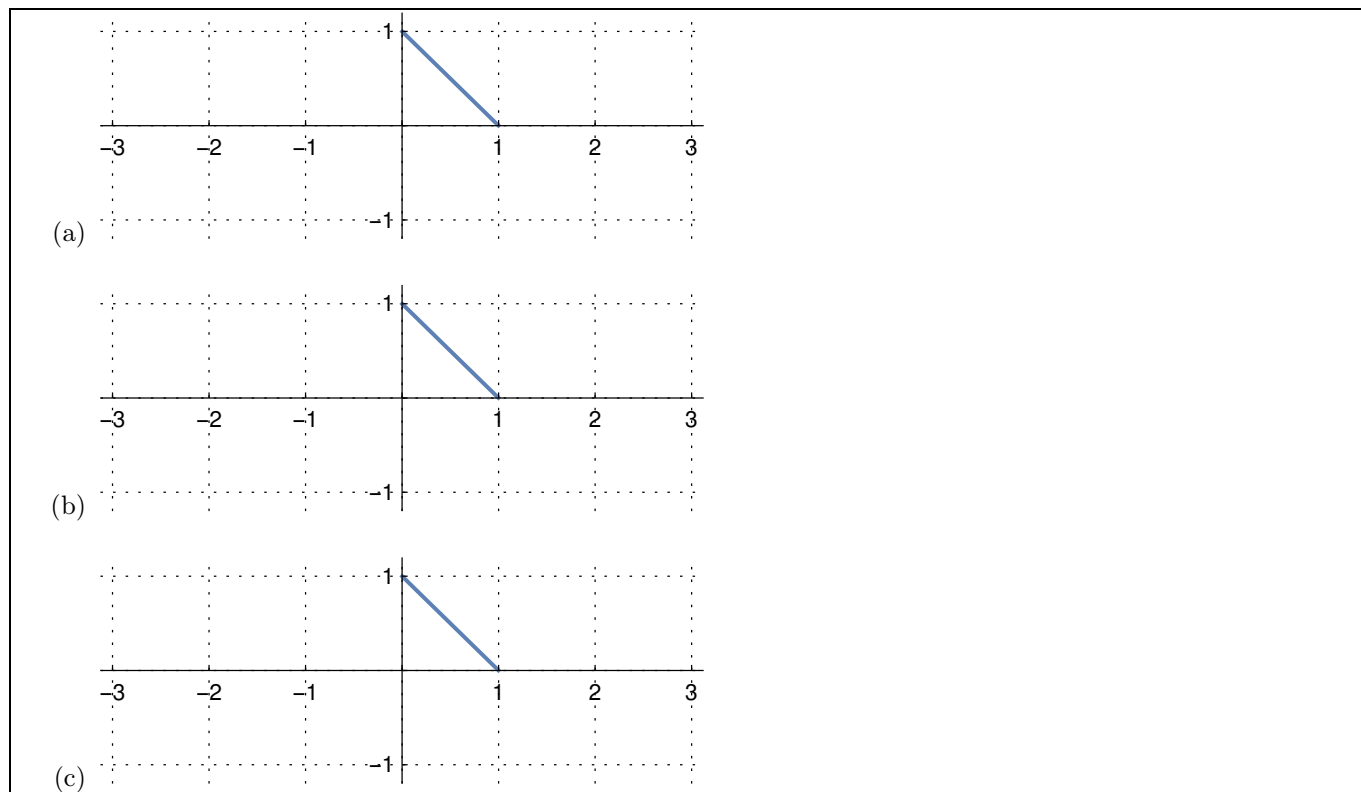
Problem 5. (3 points) Find a minimum value for the radius of convergence of a power series solution to

$$(x + 3)y'' + 2y = \frac{\cos(x)}{x^2 + 4} \quad \text{at } x = 1.$$

Problem 6. (4 points) Consider the function $f(t) = 1 - t$, defined for $t \in [0, 1]$.

- (a) Fourier series of $f(t)$ (b) Fourier cosine series of $f(t)$ (c) Fourier sine series of $f(t)$

In each sketch, carefully mark the values of the Fourier series at discontinuities.



Problem 7. (6 points)

- (a) Let y_p be any solution to the inhomogeneous linear differential equation $y'' + 5y = 1 + 2xe^{3x}\sinh(4x)$. Using the operator D write down a homogeneous linear differential equation which y_p solves.

- (b) Determine the power series around $x = 0$: $\frac{1}{1+x^2} =$

- (c) Determine the power series around $x = 0$: $2\cosh(3x) =$

- (d) Suppose $y(x) = \sum_{n=0}^{\infty} a_n(x+2)^n$. How can we compute the a_n from $y(x)$? $a_n =$

- (e) If $f(t) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi t}{3}\right)$, then we can compute the b_n as $b_n =$

(extra scratch paper)