Midterm #2

MATH 316 — Linear Algebra II
Monday, April 13, 2026

Please print your name:

No notes, calculators or tools of any kind are permitted. There are 35 points in total. You need to show work to receive full credit.

Good luck!

Problem 1. (5 points) Find the best approximation (in the L^2 sense) of $f(x) = x^2$ on the interval $[0,1]$ using a function of the form $y(x) = a\sqrt{x}$.

Problem 2. (4 points) Consider the following system of initial value problems:

$$\begin{array}{l} y_1'' = 5y_1' + 2y_2' + 4y_1 \\ y_2'' = y_1' - y_2' - 3y_2 \end{array} \qquad y_1(0) = 1,\ y_1'(0) = 4,\ y_2(0) = 0,\ y_2'(0) = 2$$

Write it as a first-order initial value problem in the form $\boldsymbol{y}' = M\boldsymbol{y}$, $\boldsymbol{y}(0) = \boldsymbol{y}_0$.

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Problem 3. (8 points) Solve the initial value problem $\boldsymbol{y}' = \begin{bmatrix} -1 & 5 \\ 1 & 3 \end{bmatrix} \boldsymbol{y}, \quad \boldsymbol{y}(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

Problem 4. (4 points) Fill in the blanks.

(a) Let A be a 4×4 matrix for orthogonally projecting onto a 3-dimensional subspace.

Then $\det(A) =$ ______, and the eigenvalues (indicate if repeated) of A are ______.

(b) If A is the 3×3 matrix for reflecting through the plane spanned by the vectors $\begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, then $A = PDP^{-1}$

with $D =$ ______ and $P =$ ______. Moreover, $\det(A) =$ ______.

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Problem 5. (6 points) Consider the sequence a_n defined by $a_{n+2} = a_{n+1} + 2a_n$ and $a_0 = 1$, $a_1 = 8$.

(a) Find an explicit (Binet-like) formula for a_n.

(b) Determine $\lim_{n \to \infty} \frac{a_{n+1}}{a_n}$.

Problem 6. (8 points) Fill in the blanks.

(a) The norm of the vector $\boldsymbol{v} = \begin{bmatrix} 2-3i \\ 1 \end{bmatrix}$ is $\|\boldsymbol{v}\| =$ ______ .

(b) If A is a projection matrix, then $A^{2026} =$ ______ . If B is a reflection matrix, then $B^{2026} =$ ______ .

(c) If A has eigenvalue 4, then $3A$ has eigenvalue ______ , A^2 eigenvalue ______ , and A^T eigenvalue ______ .

(d) If $A = \begin{bmatrix} -2 & 0 \\ 0 & 4 \end{bmatrix}$, then $A^n =$ ______ and $e^{At} =$ ______ .

(e) An example of a 2×2 matrix with eigenvalue $\lambda = 4$ that is not diagonalizable is ______ .

(f) If $N^3 = \mathbf{0}$, then $e^{Nt} =$ ______ .

(g) How many different Jordan normal forms are there in the following cases?

- A 5×5 matrix with eigenvalues $2, 2, 2, 4, 4$? ______
- A 7×7 matrix with eigenvalues $4, 4, 4, 4, 5, 5, 6$? ______

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(extra scratch paper)

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